Natural Frequency/Damping Coefficient Relationship of the Catheter-Manometer System Required for High-Fidelity Measurement of the Pulmonary Arterial Pressure

Yoshio KINEFUCHI, Toshiyasu SUZUKI, Mamoru TAKIGUCHI, Yonosuke YAMASAKI, Michio YAMAMOTO and Kunio SUWA*

Using a digital simulation method, we analyzed the relationship between natural frequency (f_n) and damping coefficient (ζ) of the catheter-manometer system required for high-fidelity measurement of the pulmonary arterial pressure. The pulmonary artery pressure waveform was obtained with a catheter-tip transducer and it was fed into a dynamic simulator programmed on a computer. The original waveform and the output of the simulator were compared and judged visually for the fidelity. From this analysis, the combination of f_n and ζ was obtained and was plotted on a $f_n - \zeta$ diagram. It showed as an area, which was convex on the left side and open on the right side. The left-convex endpoint was located at a damping coefficient of about 0.7. At a lower heart rate, this area was extended to the lower frequency side, while, at a higher heart rate, this area was limited to the higher frequency side. The $f_n - \zeta$ diagram was also constructed theoretically by calculating the relations between natural frequencies and damping coefficients of a second order system with the amplitude and phase error tolerance set at $+/-5\%$ respectively. (Key words: frequency characteristics, catheter-manometer system, natural frequency, damping coefficient, dynamic simulation)

(Kinefuchi Y, Suzuki T, Takiguchi M, et al.: Natural frequency/Damping coefficient relationship of the catheter-manometer system required for high-fidelity measurement of the pulmonary arterial pressure. J Anesth 7: 419–426, 1993)

The aim of this investigation is to analyze a frequency response of the catheter-manometer system required for accurately measuring pulmonary arterial waveform. Waves distorted on amplitude axis may lead to errors in pressure readings and waves distorted on time axis may cause undue delay when compared with other parameters such as ECG or arterial pressure. Furthermore, accuracy of depicting transient phenomena is of great importance

Department of Anesthesiology, School of Medicine, Tokai University, Isehara, Japan

**Department of Anesthesia, Faculty of Medicine, University of Tokyo, Tokyo, Japan*

Address reprint requests to Dr. Kinefuchi: Department of Anesthesiology, Tokai University, School of Medicine, Bohseidai, Isehara, Kanagawa, 259-11 Japan

as in pulmonary artery occlusion to estimate pulmonay capillary pressure. Usually, such analysis is performed on individual catheter-manometer systems experimentally. If we can define this system in a general term, we should be able to analyze this theoretically on general terms.

Experimental results of the frequency characteristics of the catheter-manometer system for frequencies up to 30 Hz indicate the following characteristics[1–3]. The curves are flat in the lower frequency range. A so-called resonant phenomenon is observed in a frequency range of 8 to 14 Hz. In the higher frequency range it declines at a rate of −12 dB/octs. These characteristics show that the system is of a simple second order.

A second order system applied to a catheter-manometer system consists of the catheter's elastic recoil and inertial (mass) and frictional properties of its internal fluid. These individual components then determine two parameters; the natural frequency (f_n), and the damping coefficient (ζ). The last two parameters are measurable and strictly characterize the dynamic response of any catheter-manometer system[4–6].

We, therefore, studied the relations of natural frequency and damping coefficient and developed a simulated catheter-manometer system within a computer. Using this simulator, we compared the original waveform and the waveform through the catheter-manometer system when two parameters were varied.

Materials and Methods

(1) Dynamic simulation: The pressure applied to the tip of the catheter equilibrates with 3 forces, i.e., the inertial force, the elastic force and the frictional force as mentioned above. The kinetic equation of this type may be given by the following general expression[4–8]:

$$1/(\omega_n)^2 \mathrm{d}^2\mathrm{x}/\mathrm{dt}^2+(2\zeta_n)\mathrm{dx}/\mathrm{dt}+\mathrm{x}=\mathrm{p}$$

In the above equation, ω_n denotes the natural frequency expressed with angular frequency, and ζ, the damping coefficient. The frequency is given as f, $\omega_n = 2\pi f_n$. p is the original (true) pulmonary arterial pressure waveform and x is the waveform obtained through the catheter manometer system. Measurement of the pulmonary arterial pressure waveform is dependent only on the values of ω_n (or f_n) and ζ.

Runge Kutta algorithm[9–11] is constructed on a computer (PC98RL; NEC, JAPAN) and applied in solving this differential equation. The time intervals for the algorithm were 2 msec. The ranges studied were 3–60 Hz for natural frequency, and 0.05–2.0 for the damping coefficient, respectively. The pulmonary arterial pressure waveform was recorded digitally on the computer after AD conversion. The calculation was later performed yielding the value of x in the above equation.

(2) Frequency analysis of pulmonary arterial pressure: Six mongrel dogs, ranging from 10 to 14 kg in body weight, were used for this part of the study. Anesthesia was induced with 20 $\mathrm{mg{\cdot}kg^{-1}}$ of pentobarbital and maintained with nitrous oxide and oxygen. A catheter-tip transducer having the frequency bandwidth of DC to 1 Kz (PT-157J; Goodtec, U.S.A.) was inserted into the pulmonary artery through the femoral vein.

Tachycardia with a heart rate of 150+/−5 bpm was produced by administration of 0.5 mg atropine sulfate, and bradycardia with a heart rate of 90+/−5 bpm was produced by administration of 2 mg neostigmine. At their respective stable heart rate, the waveforms from the catheter-tip transducer were recorded on a magnetic data recorder (A67; SONY, JAPAN). A typical waveform was chosen for each heart beat, which was converted

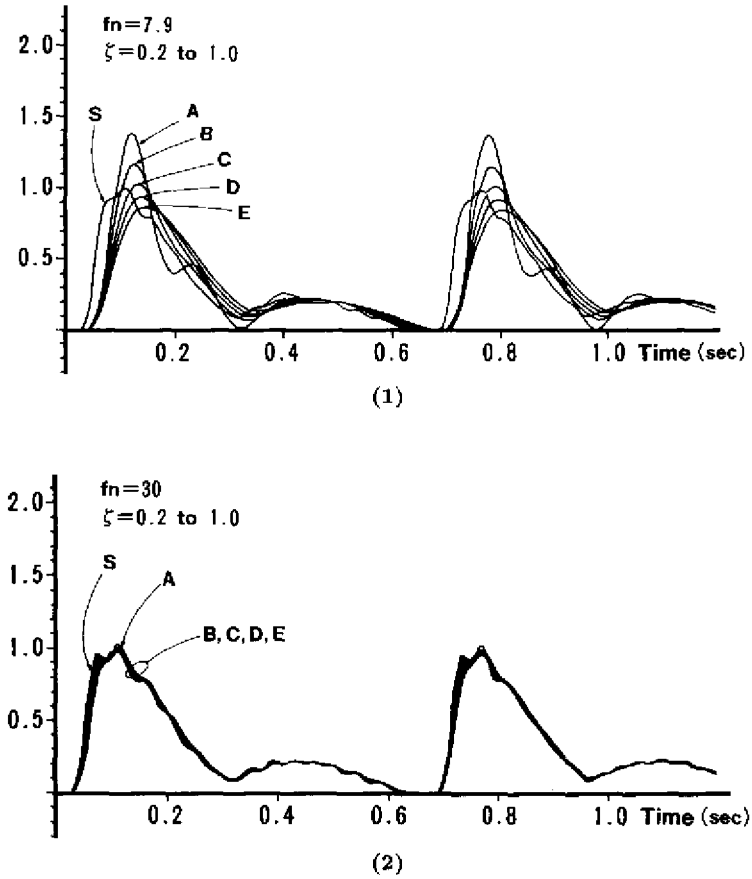

Fig. 1. Changes in pulmonary artery pressure waveform.

The degrees of change in the pulmonary arterial pressure waveform at a heart rate of 91 bpm (1.5 Hz) on passing through a simulator (1) at 7.9 Hz (natural frequency), and (2) at 30 Hz are shown. The damping coefficient is varied in the range from 0.2 1.0 (A-E in the figure) against the original waveform of pulmonary artery pressure directly obtained (S in the figure). See text for details.

into the digital data file through an AD converter having the precision of 12 bits conversion (AnalogProII; Canopus Electronics, JAPAN).

Frequency analysis of the waveform at each heart rate was performed by use of the fast Fourier transform algorithm on the above-mentioned computer. Within the sampling intervals of 5 msec, the data number of 1024, and the duration of each sampling time of 5.1 sec, the ensemble mean of 6 repeats of measurement over about 31 consecutive seconds was taken. The error components due to data sampling were corrected with cosine-taper window.

For obtaining the relationship between natural frequency and damping coefficient required for high-fidelity measurement, we performed the following procedure. We fixed the natural frequency at certain frequency and adjusted the damping coefficient so that no discernible differences between simulated and original waveforms are obtained. Thus we obtained one pair of natural frequency and damping coefficient. We then varied the natural frequency and found the corresponding damping coefficient. With this procedure, a range of combination of natural frequency and damping coefficient was constructed. Heart rates of 154 bpm (2.5 Hz) and 91 bpm (1.5 Hz) were chosen. The following criteria were applied for evaluating the fidelity of the waveform: (a) wave heights that were consistent within a tolerance of +/−5%; (b) a negligible or no oscillation due to under-damping; and (c) uniform time

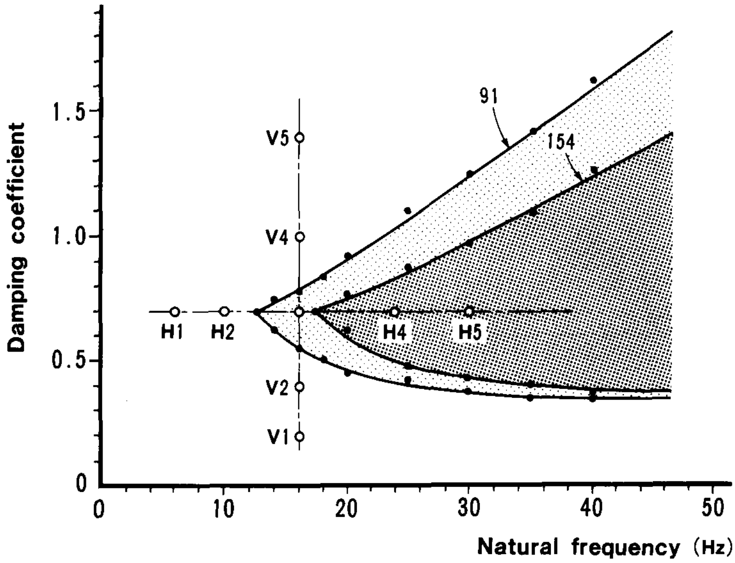

Fig. 2. Adequate frequency response area given by natural frequency and damping coefficient ($fn - \zeta$) diagram).

Two areas convex on the left and open on the right side marked as 154 and 91 show the adequate frequency response areas where high-fidelity waveforms can be obtained at the respective heart rate. The natural frequencies must be 17.5 Hz or higher at 154 bpm, and 12.5 Hz or higher at 91 bpm, and in the higher frequency range, the range of damping coefficient to be adjusted gradually becomes wider. See text for details.

lag over the entire waveform.

Results

(1) Relations of waveform distortion to the natural frequency and damping coefficient by dynamic simulation: Figure 1 shows a plot of some of the simulated distortion data at a heart rate of 91 bpm (1.5 Hz). In figure 1(1) the natural frequency was set at 7.9 Hz, with the damping coefficient varied in the range from 0.2 to 1.0 (A-E in the figure). With a smaller damping coefficient, "overshoot" in the waveform, as well as an occurrence of oscillation or ringing were observed. With a larger damping coefficient, the waveform became smoothed out, and high frequency components were lost, associated with a large time lag in the entire waveform. At a damping coefficient of about 0.6, the optimal waveform was obtained, giving a correct peak pressure value, but there were discernible differences between the two waveforms. In other words, it is obvious that with the natural frequency set at 7.9 Hz, it is impossible to obtain a waveform with fidelity, even if the damping coefficient is adjusted to an optimal setting.

Figure 1(2) represents another example at a natural frequency of 30 Hz. With a damping coefficient of 0.2 (A in the figure), a small overshooting occurred, but the waveform was associated with a minimal time lag and with excellent details of waveform. Figures 1(1) and 1(2) show that with higher natural frequency of the system, the details of the original waveform were maintained accurately. A waveform with the highest fidelity was obtained with a damping coefficient of about 0.6. As for the time lag, the smaller the damping coefficient, the smaller was the time lag of the entire waveform.

The range of combination of natural frequency and damping coefficient required for high-fidelity measurement is shown in figure 2. In the figure, the abscissa represents the natural frequency, and the ordinate, the damping coefficient; two areas convex on the left side and open on the right side, and marked as 154 and 91 represent the adequate frequency response areas at the respective heart rates. The lowest natural frequencies were 17.5 and 12.5 Hz respectively, when the damping coeffi-

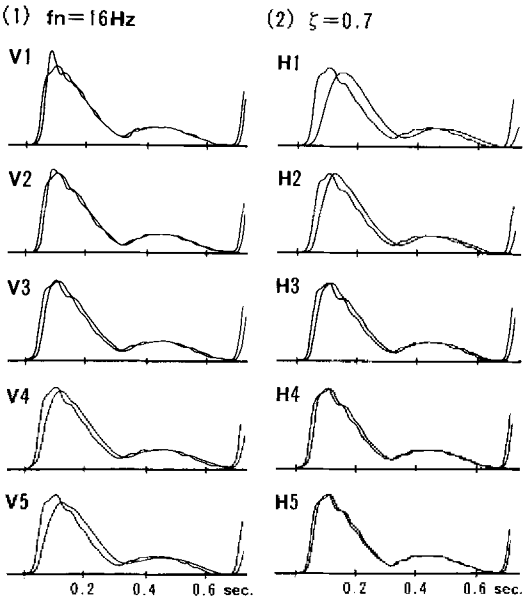

Fig. 3. Changes in simulated waveforms on the $fn - \zeta$ diagram.

The simulated waveforms at the points marked V1–V4 in figure 2 are given in (1); and those marked H1–H4, in (2). The higher the natural frequency, the higher fidelity waveform is reproduced, and the highest is obtained at a damping coefficient of about 0.7, and the higher the natural frequency, and also the lower the damping coefficient, the time lag becomes smaller.

cient had to be adjusted to about 0.6–0.7. The allowance of damping coefficient becomes gradually greater with the higher natural frequencies.

Figure 3(1) shows the examples of simulated waveform at V1–V4 in figure 2. At any natural frequency, a damping coefficient of 0.6–0.7 yielded a waveform of the highest fidelity. As for the time lag, however, the lower the damping coefficient, the smaller the time lag. Figure 3(2) shows the results of simulation at H1–H4 in figure 2. At a fixed damping coefficient, the higher the natural frequency, the better the waveform fidelity and the smaller the time lag.

(2) Frequency components of pulmonary artery pressure waveform figure 4 shows the frequency components of pulmonary artery pressure waveform at the heart rates of 154 bpm (2.5 Hz) and 91 bpm (1.5 Hz). The frequency components were described as the harmonics of the basic or fundamental heart rate. In the figure, the power of each harmonic is normalized with the power of fundamental harmonic corresponding to the heart rate as 1. If the harmoncis corresponding to 5% or more of the power of fundamental harmonic are taken as effective waveform components, they are distributed to 12.5 Hz and 7.5 Hz, which correspond to the fifth harmonic, respectively. The pressure waveform varies greatly with the status of circulation, associated with changes in the frequency components, but the power of those harmonics above the fifth is small and mostly falls within the basic noise level of the system. This finding is consistent with those reported by Gersh[8], Patel et al.[12], and Milnor[13]. The frequency band-width of pulmonary artery pressure waveform may therefore be taken as about 5 times the heart rate, which is also a band-width required for a catheter-manometer system. The relations of the required band-width of catheter-manometer system to the 2 parameters described in the preceding section are discussed later.

Discussion

The solution of the second-order system and its dynamic simulation is feasible by building a specific circuit network with an analog computer or operational amplifiers[5,14]. While such analogue technique may be suitable for real-time operation, it is less so when subsequent analysis is planned. In our analysis, we needed frequency analysis and secondary data processing related to natural frequency and damping coefficient. We, therefore, employed the digital operation using the Runge-

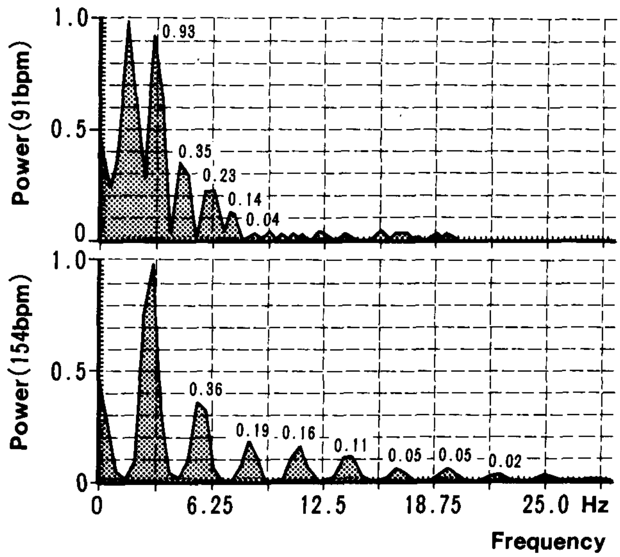

Fig. 4. Distribution of frequency components of pulmonary artery pressure waveform.

The frequency components of pulmonary artery pressure waveform at the heart rates of 154 bpm (2.5 Hz) and 91 bpm (1.5 Hz) are shown. The power of each harmonic is normalized with the power of fundamental harmonic corresponding to heart rate as 1. The harmonics corresponding to 5% or larger of the power of fundamental harmonic are distributed to 12.5 and 7.5 Hz, corresponding to 5 times the heart rate.

Kutta method. The operation speed on a small computer is rather slow, but it is not an essential problem involved in this theme. It is possible to speed up the operation by electing a more adequate program language and implementing a device with a high-speed operation.

Figure 2 is similar to that reported by Gardner on the relations of arterial pressure waveform to natural frequency and damping coefficient[14]. Using the arterial pressure waveform as an objective, he produced 2 adequate frequency response areas at the mean heart rates of 94 bpm and 118 bpm by use of an analog simulator. As we did, he visually judged the quality of simulated waveforms, although he did not clearly state his criteria. When his area at a heart rate of 94 bpm is compared with our area at a close heart rate, i.e., 91 bpm, his adequate frequency response area is lower in frequency of about 5.5 Hz. The discrepancies between our findings and his may have derived either from the difference in the criteria used or from the difference in the frequency band-width of the original waveform. In any event, it is unavoidable that the range of adequate frequency response area varies with heart rate and waveform.

Up to this point, we judged the fidelity on visual pattern. This is subjective and is bound to vary from one researcher to the other. We may use a more scientific, objective criteria for the fidelity. For this purpose, we chose the error on frequency-amplitude and frequency-phase response. We set the error tolerance to be +/−5% and then calculated theoretically the combination of natural frequency and damping coefficient accordingly. Figure 5 represents the results. In the figure, f_h denotes the highest frequency at which the wave may be accurately reproducible. As shown in figure 4, the band-width extends to the frequency of 5 times the heart rate, and which corresponds to f_h. This means that the heart rate (in bpm) equivalent to f_h equals 60 times of f_h divided by 5. HR in the parenthesis shows this equivalent heart rate. For example, a heart rate of 91 bpm equals 1.5 Hz and a required band-width for this rate is 7.5 Hz, which is the f_h for this condition. We do not have a line for f_h of 7.5 in

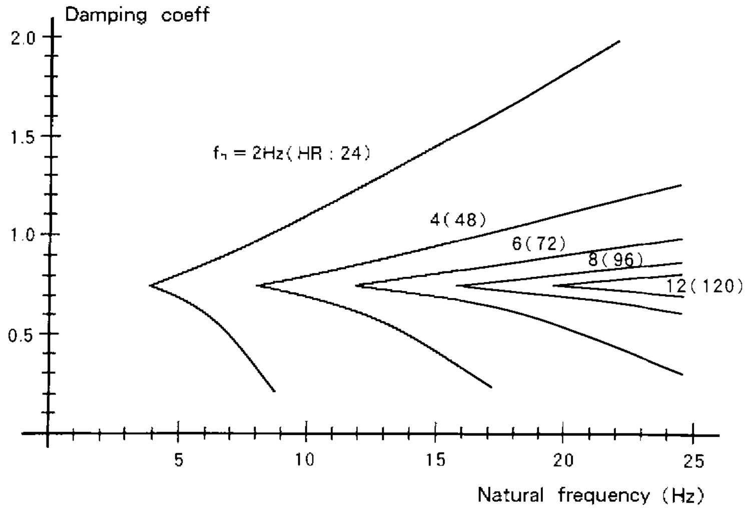

Fig. 5. Adequate frequency response area calculated theoretically with the error tolerance set at +/−5% ($f_n - \zeta$ diagram).

This figure indicates the area where adequate frequency response may be obtained. The relationships between the damping coefficient (ζ) and natural frequency (f_n), and the f_h, the highest frequency obtainable within +/−5% error, are shown on the graph for f_h values of 2, 4, 6, 8 and 10 Hz. HR in the parenthesis equals 60 times of f_h devided by 5, and means the highest heart rate corresponding to f_h (see txt for details). The combination of the damping coefficient and natural frequency of the measuring system should be on the right side of the corresponding f_h line. In other words, the f_h line set the highest frequency at which the wave may be accurately reproducible.

figure 5, but by interpolation we obtain the coresponding natural frequency being around 14 Hz, the lowest frequency point on the line for f_h of 7.5. This area is higher in frequency by about 1.5 Hz than that given in figure 2, and higher by about 7 Hz than that given by Gardner[14]. Although it is not definitive whether the tolerance of +/−5% of figure 5 is adequate as criteria for judgment, the results given in figure 2 are close to this area, while Gardner's area appears too broad. This clearly shows that the adequate frequency response area always depends on the frequency range of original waveform. We insists that it should be accompanied by the explicit statement of the frequency range of original waveform. Alternatively the heart rate should be specified at least as a substitute for the former. In this sense, the adequate frequency response area shown in figure 5 is most universal.

The frequency characteristics of the catheter manometer system has been evaluated on the adequate frequency response area created by the technique of dynamic simulation. On its application, we may reverse the process. From the waveform of the catheter-manometer system, we may reproduce the original waveform by passing the signal through a filter, which corrects or compensates for the frequency dependence of the catheter-manometer system. This study established the basic ground on which the operation of such reverse transfer function is applied.

Example: If the damping coefficient is 0.7 and natural frequency is 10 Hz, then the system is adequate for measuring up to 2 Hz (the point is located to the right to f_h of 2 Hz line), but inadequate for measuring up to 6 Hz (the point is located to the left to f_h of 6 Hz line). For more than 0.7, the system is overdamped. The required natural frequency for the same f_h increases. For less than 0.7, the system is underdamped. The requried natural frequency for the same f_h again increases. For obtaining these lines for f_h values, see Appendix.

Appendix: The procedure for obtaining lines in figure 5 is as follows. First, for each damping coefficient, we defined the frequency range or ranges in which both amplitude- and phage-error are within +/−5% on Bode diagram. This information is then transformed to the relationship between f_h-natural frequency and damping coefficient of the system. The results were plotted on the axis of damping coefficient (ordinate) and natural frequency (abscissa) as figure 5. The actual calculation is done by a complex computer software developed by the authors.

(Received May 1, 1992, accepted for publication Jan. 6, 1993)

References

1. Shinozaki T, Deane RS, Mazuzan JE: The dynamic response of liquid-filled catheter systems for direct measurements of blood pressure. Anesthesiology 53:498–504, 1980
2. Rutten AJ, Nancarrow C, Ilsley AH, et al: An assessment of six different pulmonay artery catheters. Crit Care Med 15:250–255, 1987
3. Hipkins SF, Ruten AJ, Runciman WB, et al: Experimental analysis of catheter-manometer systems *in vitro* and *in vivo*. Anesthesiology 71:893–906, 1989
4. Fry DL: Physiologic recording by modern instruments with particular reference to pressure recording. Physiol Rev 40:753–788, 1960
5. Milhorn HT: Transient analysis of physical systems, the application of control theory to physiological systems. Philadelphia, WB Sasunders, 1966, pp. 78–93
6. Kleinman B: Understanding natural frequency and damping and how they relate to the measurement of blood pressure. J Clin Monit 5:137–147, 1989
7. Milnor WR: Methods of measurement, hemodynamics. Baltimore, Williams & Wilkins, 1982, pp 272–301
8. Gersh BJ: Measurement of intravascular pressures, the circulation in anaesthesia: Applied physiology and pharmacology. Edited by Prys-Roberts C. Oxfor, Blackwell Scientific Publications, 1980, pp 511–518
9. Jaeger JC, Starfield AM: Differential equations and the numerical solution of differential equations, an introduction to applied mathematics. London, Oxford University Press, 1974, pp 407–430
10. Verner JH: Explicit Runge-Kutta methods with estimates of the local truncation error. SIAM J Numer Anal 15:772–790, 1978
11. Thomas B: The Runge-Kutta methods, approximating solutions to ordinary differential equations. BYTE: 191–210, April, 1986
12. Patel DJ, Mason DT, Ross J, et al: Harmonic analysis of pressure pulses obtained from the heart and great vessels of man. Am Heart J 69:785–794, 1965
13. Milnor WR: Pulsatile blood flow. N Engl J Med 287: 27–34, 1972
14. Gardner RM: Direct blood pressure measurement-dynamic response requirements. Anesthesiology 54:227–236, 1981